must be taken of all the recoil momenta from the initial cascade and from particle evaporation both before and after scission. A few typical paths for producing Na^{24} were tested by Monte Carlo calculation. Energies were found to be in rough agreement with the observed mean, and the values are not very sensitive to whether the scission takes place near the beginning or near the end of the evaporation phase. The wide distribution of observed energies probably cannot be accounted for only by the different ways in which the various velocity vectors couple with each other. Another important factor may be the variability of nuclear deformation prior to scission, even for the formation of the same products. Relatively minor broadening of the energy spectra arises from the circumstance that each product may result from a variety of parent nuclei and by β decay of various precursors.

We can now see why one should not expect a very substantial difference between the energy spectrum for a given product formed by the spallation mechanism and the spectrum for that product formed by a two body breakup mechanism. In the first case, the velocity imparted to the nucleus during the prompt cascade followed by evaporation of nucleons, alpha particles, and heavier particles, serve to displace the spectrum to higher energies. In the second case, the spectrum is displaced to lower energies from the nominal Coulomb energy by the processes discussed above.

The data of the present experiment are insufficient in themselves to determine whether the Na^{24} is formed from silver by a fast fragmentation mechanism or by a slower fission type mechanism. The fast mechanism is supported by the thick-target recoil experiments of Crespo, Alexander, and Hyde[3] while the slow mechanism is supported by the nuclear emulsion data of Baker and Katcoff.[12] It would be interesting to extend the present experiment for Na^{24} by making range measurements at forward and backward angles also, as was done recently by Cumming *et al.*[4] for Na^{24} produced from Bi. Their experiments showed that in this instance the Na^{24} was produced by a rapid process.

ACKNOWLEDGMENTS

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Comparison Between Reactions of Alpha Particles With Scandium-45 and Deuterons with Titanium-47†

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The excitation functions for the (α,n), $(\alpha,2n)$, $(\alpha,2p)$, $(\alpha,\alpha'n)^m$, $(\alpha,\alpha'n)^g$, and $(\alpha,2pn)$ reactions of Sc^{45} and the (d,n), $(d,2n)$, $(d,2p)$, $(d,\alpha n)^m$, and $(d,\alpha n)^g$ reactions of Ti^{47} were measured for alpha-particle energies from 15 to 40 MeV and for deuteron energies from 4 to 20 MeV. The alpha-particle excitation functions can be successfully fitted by a calculation based upon compound-nucleus theory; but the calculation is less successful in reproducing the deuteron excitation functions. The divergences between calculation and observation are in a direction that is to be expected from the contribution of deuteron stripping reactions. Despite the indications that the deuteron-induced reactions have substantial noncompound contributions, the ratio of the cross sections for the $(\alpha,2p)$ and $(\alpha,2n)$ reactions have the same dependence upon the excitation energy of the compound system as has the corresponding ratio for the $(d,2p)$ and $(d,2n)$ reactions.

I. INTRODUCTION

MANY investigations have been carried out on nuclear reactions induced by protons and alpha particles in the medium weight elements ($45<A<75$) at energies up to a few tens of MeV. Dostrovsky *et al.*[1] have succeeded, to a large extent, in fitting all the existing excitation functions with the compound-nucleus model. Implicit in this model is the assumption made by Bohr in 1937 that the modes of decay of the compound nucleus are independent of the modes of formation. To

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[1] I. Dostrovsky, Z. Fraenkel, and G. Friedlander, Phys. Rev. 116, 683 (1959).

date only four studies,[2] performed with protons and alpha particles, have been made to provide experimental verification of this assumption. The results of these experiments supply qualitative support to the assumption; but there are quantitative disagreements which are evidently outside of experimental error. Thus, despite the successes of compound nucleus theory in fitting excitation functions, the question still remains as to whether or not these successes are significant, or merely a consequence of the several parameters available within the theory.

To explore this question further, we have measured excitation functions for reactions between alpha particles and Sc^{45} and between deuterons and Ti^{47}. The latter reactions are expected to proceed, at least partly, through noncompound processes. Thus, in an extreme instance, it may be seen whether or not noncompound processes may easily be distinguished from compound processes in this type of study.

Specifically, the investigation consisted of the following four parts:

(1) The excitation functions of the reactions induced by alpha particles on scandium-45 were measured. The kinetic energy of the alpha particles ranged from 15 to 40 MeV.

(2) Cross sections for the (α,n), $(\alpha,2n)$, $(\alpha,2p)$, and $(\alpha,\alpha'n)$ reactions were compared with computed values based on evaporation theory.

(3) The excitation functions of the reactions induced by deuterons on titanium-47 were measured. The kinetic energy of the deuterons ranged from 4 to 20 MeV. Since the same compound system, vanadium-49, is formed in both reactions, $\alpha+Sc^{45}$ and $d+Ti^{47}$, the data from these two reactions were compared directly in an experimental test of Bohr's assumption.

(4) The divergences from Bohr's assumption that were found were used to estimate the relative importance of stripping and compound nucleus reactions for deuterons.

II. EXPERIMENTAL PROCEDURES

A. Bombardments

Stacks of target foils, interspaced with aluminum absorbers to degrade the energy of the beam, were bombarded in the deflected beam of the Brookhaven 60-in. cyclotron. Targets were placed so that recoiling products were caught in the aluminum backing of the targets. The beam intensity was measured to within $\pm 5\%$ with a Faraday cup.[3] The incident energy of the alpha beam was determined to within ± 0.2 MeV by the gross activity of copper foils placed in the stack.[4] With the range-energy curves of Aron *et al.*[5] the energy of the alpha beam at various positions in the stack was determined. In order to determine the energy degradation of the deuteron beam, a range-energy table for the deuterons was constructed from range-energy tables by Bichsel.[6] The incident energy of the deuteron beam was determined by the measurement of its range in aluminum. The bleaching of thin foils of blue cellophane by the deuterons at the end of their range provided a sensitive method for measurement of the range.[7]

B. Target Foils

Scandium targets were made by depositing uniform layers of scandium oxide on very pure (99.99%) 0.001-in.-aluminum foils by means of a Zapon painting technique.[8] The average thickness of the targets was of the order of 100 μg per cm^2. Targets were uniform to within 1–2% as indicated by chemical analysis and β-backscattering measurements.

Titanium targets were made by settling suspensions of titanium dioxide in alcohol onto the pure aluminum foils. It was necessary to purify the enriched titanium dioxide from Oak Ridge National Laboratory by dissolving it in HF, evaporating the HF solution with H_2SO_4, and precipitating titanium hydroxide with ammonia. The settling time of the suspensions varied from one to two days. After the alcohol had evaporated, the targets were dried in an oven at 100°C and covered with the pure aluminum foils to prevent damage to the uniformity of the deposit either due to jarring or rubbing during irradiations. The average thickness of the targets was of the order of 200 μg per cm^2. The targets made by this method were found to be uniform to within $\pm 5\%$ by chemical analysis.

C. Chemical Separations

Chemical separation procedures were essentially identical for both the scandium and titanium targets. After irradiations, each target was dissolved in a mixture of HCl, HNO_3, and known amounts of scandium and vanadium carrier solutions. For the titanium targets, an aliquot of this solution was analyzed for titanium content. The solutions were adjusted to pH 2 and scandium was then extracted into a benzene solution of TTA (thionyl trifluoracetate) leaving aluminum, vanadium, and titanium in the aqueous phase. Scandium was back-extracted, precipitated, and deposited in a defined area.

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[2] S. N. Ghoshal, Phys. Rev. **80**, 939 (1950); W. John Jr., *ibid.* **103**, 704 (1956); C. M. Stearns, Ph.D. thesis, Columbia University, New York, 1961 (unpublished); S. Tanaka, M. Furakawa, S. Iwata, M. Yagi, H. Amano, and T. Mikumo, J. Phys. Soc. Japan **15**, 2125 (1960).

[3] S. Amiel and N. T. Porile, Rev. Sci. Instr. **29**, 1112 (1958).

[4] N. T. Porile and D. L. Morrison, Phys. Rev. **116**, 1193 (1959).

[5] Aron, Hoffman, Williams, U. S. Atomic Energy Commission Report, AECU-663, May 1951 (unpublished).

[6] H. Bichsel, Phys. Rev. **112**, 1089 (1958).

[7] J. B. Cumming (private communication).

[8] R. W. Dodson, A. C. Graves, L. Helmholz, D. L. Hufford, R. M. Potter, and J. G. Povelites, in *Miscellaneous Physical and Chemical Techniques of the Los Alamos Project*, edited by A. C. Graves and D. K. Froman (McGraw-Hill Book Company, Inc., New York, 1952), p. 1.

For scandium targets, the precipitate was Sc 8-hydroxyquinolate; for titanium targets, the precipitate was $Sc(OH)_3$.

Vanadium was precipitated from an acid solution as the sulfide which was then deposited.

D. Determinations of Disintegration Rates

Gamma spectroscopy was used wherever possible to identify the radioactive isotopes and to measure their disintegration rates. With the exception of V^{48} from the scandium targets, the activities of all isotopes were measured on a NaI(Tl) scintillation-crystal photomultiplier assembly connected to a 100-channel pulse-height analyzer. All scintillation-counting data were corrected for γ absorption, Compton background, and sum peaks. V^{48} activities from the scandium targets were counted on β-proportional counters. Measured activities for all isotopes were converted to absolute cross sections by making appropriate corrections for chemical yields, saturation effects during bombardment, and detector efficiencies.

E. Samples from Scandium Targets

Two NaI(Tl) crystals (3×3 in. and $1\frac{1}{2}$×1 in.) were calibrated with a standard Na^{22} sample whose absolute disintegration rate was determined to within ±4%. The measured areas under the 0.511- and 1.276-MeV photopeaks of the Na^{22} standard gave two points on a photopeak efficiency (PE) versus energy (E) curve. It has been shown[9] that on a log-log plot, intrinsic peak efficiency (IPE) versus energy (E) curve is a straight line from 0.3 to 2 MeV. The PE versus E curve is expected to behave in the same manner since PE=IPE ×geometry factor, and the samples were counted in a fixed geometry. The straight line portion of the curve was verified by counting the 100% abundant 0.885- and 1.12-MeV gamma rays of Sc^{46}; the experimental ratio of the photopeaks of these two gamma rays agreed within 1% with the ratio of the photopeak efficiencies obtained from the straight line interpolation. The PE versus E curve for the thin crystal was extended down to 0.160 MeV by counting the 0.270-MeV gamma ray of Ba^{135} and the 0.140-MeV gamma ray of Mo^{99} standards. The maximum error in the PE of the 0.160-MeV gamma ray on the thin crystal is ±10%.

Radiations from V^{48} samples were counted on β-proportional counters which were calibrated by a vanadium sample standardized through its 100% abundant 0.986-MeV gamma ray. Samples were sufficiently thin so that self-absorption effects were negligible.

F. Samples from Titanium Targets

The gamma rays from all samples were counted by a 3×3-in. NaI(Tl) crystal except for those from Sc^{47} which were counted on the 6 mm NaI(Tl) crystal. Calibration curves of photopeak efficiency (PE) versus energy (E) for these two crystals were obtained by counting standardized samples of Mn^{54}(0.840-MeV γ), Zn^{65}(1.12-MeV γ), Sr^{85}(0.513-MeV γ), Hg^{203}-(0.279-MeV γ), and Am^{241}(0.060-MeV γ). The maximum error in the PE versus E curve is approximately 5% for both crystals.

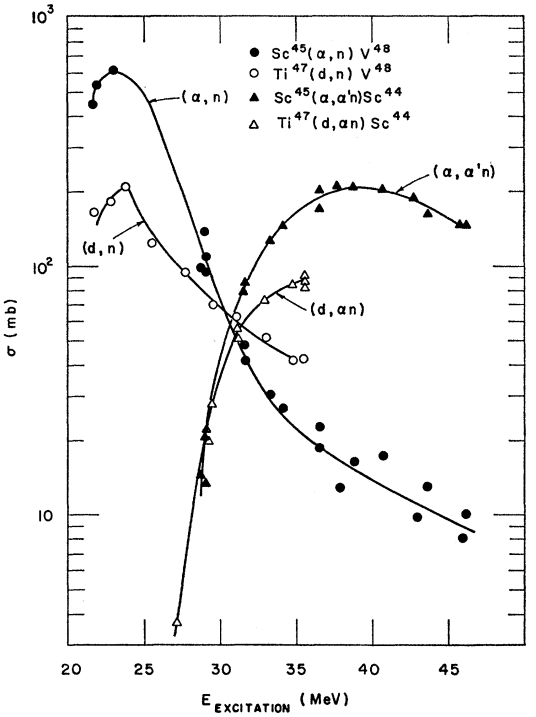

FIG. 1. Excitation functions for the (d,n), (α,n), $(d,\alpha n)$, and $(\alpha,\alpha' n)$ reactions.

III. RESULTS

Cross sections for various reactions between alpha particles and Sc^{45} nuclei are given in Table I and plotted in Figs. 1–4. The second column of the table gives the laboratory energy of the incident particle; the third gives the excitation energy of compound system. Errors in the cross-section measurements arise mainly in the determination of detection efficiencies. For $Sc^{44,44m}$, Sc^{46}, V^{47}, and V^{48}, for which the gamma rays and annihilation radiation were counted on the 3×3 in. and the 6-mm NaI(Tl) crystals, the errors in the counting efficiencies are estimated to be approximately 5%. For Sc^{47} samples counted on the $1\frac{1}{2}$×1-in. NaI(Tl) crystal, the uncertainty in the counting efficiency of the 0.160-MeV gamma ray may be as large as 10% because of the lack of appropriate gamma-ray standards in this energy range. For those V^{48} samples counted on β-proportional counters, the error in the gamma efficiency and the error in beta efficiency due to the neglect of scattering effects for the individual samples lead to a

[9] N. H. Lazar, R. C. Davis, and P. R. Bell, Nucleonics **14**, 52 (1956).

TABLE I. Cross sections for alpha particles with scandium-45 (mb).

Run No.	E_α(MeV)	E_{ex}(MeV)	$\sigma(\alpha,n)$	$\sigma(\alpha,2p)$	$\sigma(\alpha,2n)$	$\sigma(\alpha,\alpha'n)^m$	$\sigma(\alpha,\alpha'n)^g$	$\sigma(\alpha,\alpha'n)^{m+g}$	$\sigma(\alpha,2pn)$
KL-3	40.1	46.0	10.1			90.4	56.4	146	95.1
	34.3	40.7	17.1						
	29.7	36.5	22.4			94.8	74.0	168	
	24.5	31.7	58.5			46.1	38.6	84.7	
	21.5	29.0	138			6.6	6.8	13.5	
KL-4	37.4	43.6	13.0			105	55.2	160	81.7
	32.2	38.8	16.1			132	74.6	206	30.6
	27.2	34.2	27.0			89.9	55.5	145	
	21.6	29.1	109			13.8	8.2	22.0	
	15.1	23.0	618						
KL-5	39.9	45.8	7.9	18.8		97.6	51.1	148	99.7
	34.2	40.6		31.4		124	77.7	201	53.0
	29.7	36.5	18.6	33.4		114	84.8	198	
	24.4	31.6	41.7	17.9		45.3	34.4	79.7	
	21.5	29.0	94.0			12.9	8.8	21.7	
KL-6	36.6	42.8	9.7	29.2		122	60.8	182	72.2
	31.2	37.8	12.8	39.2		130	79.7	209	22.0
	26.2	33.2	30.2	24.8		74.1	50.3	124	
	21.2	28.6	98.6	10.2		8.5	5.7	14.2	
	13.5	21.6	453						
KL-7	36.3	42.5			71.1				
	31.5	38.1			141				
	26.5	33.5			196				
	20.8	28.3			139				
KL-8	39.5	45.5			53.2				
	33.7	40.2			109				
	29.0	35.9			180				
	23.7	31.0			192				
KL-9	37.1	43.3			63.3				
	31.9	38.5			143				
	26.9	33.9			202				
	21.4	28.8			160				
KL-10	39.9	45.9			45.1				
	34.2	40.6			100				
	29.6	36.4			168				
	24.3	31.6			192				
KL-11	18.7	26.4			95.0				
	16.8	24.6			51.4				
	14.7	22.7			8.2				
KL-5	13.7	21.8	535						

cumulative error in the efficiency of approximately 15%.

Since the TiO_2 used in the deuteron studies was not enriched to 100% in Ti^{47} (see Table II), the presence of other isotopes can lead to products which are also produced from Ti^{47} (see Table III). Therefore, excitation functions for reactions leading to these products were measured for all titanium isotopes as shown in Table IV. The cross sections reported there are based upon an assumed 100% enrichment. Then, from these cross sections and from the known isotopic abundances of the enriched samples, simultaneous equations were solved to obtain cross sections for a particular isotope. The corrected cross sections for reactions between deuterons and Ti^{47} are tabulated in Table V and plotted in Figs. 1–3; those for the $Ti^{46}(d,\alpha)Sc^{44,44m}$ are given in Table VI and plotted in Fig. 5. The 20% scatter of the measured cross sections for the $(d,2n)$ reaction about their mean values is not understood; the smooth curve shown in Fig. 2 was obtained by averaging the smoothed results from the five determinations.

Corrections due to recoil losses were not necessary because the aluminum backings of the targets which stopped the recoil products were always dissolved along with the targets.

The cross sections of $Ti^{48}(d,\alpha)Sc^{46}$ and $Ti^{46}(d,\alpha)Sc^{44,44m}$ at 7.7-MeV deuteron energy agreed within 25% with those obtained by Anders and Meinke.[10] The cross sections of $Ti^{48}(d,2n)V^{48}$ agreed within 10–20% with those obtained by Burgus *et al.*[11]

[10] O. U. Anders and W. N. Meinke, Phys. Rev. **120**, 2114 (1960).
[11] W. H. Burgus, G. A. Cowar, J. W. Hadley, W. Hess, T. Shull, M. L. Stevenson, and H. F. York, Phys. Rev. **95**, 750 (1954).

TABLE II. Composition of enriched titanium isotopes.

Enriched Sample	Isotope (Ti)	Abundance (%)
Ti^{46}	46	86.1
	47	1.6
	48	10.6
	49	0.8
	50	1.0
Ti^{47}	46	1.7
	47	85.6
	48	11.3
	49	0.8
	50	0.6
Ti^{48}	46	0.163
	47	0.316
	48	98.90
	49	0.479
	50	0.143
Ti^{49}	46	1.3
	47	1.3
	48	14.5
	49	81.5
	50	1.4
Ti^{50}	46	2.51
	47	1.62
	48	12.23
	49	3.33
	50	80.31

IV. DISCUSSION

Before proceeding to a quantitative treatment of these results, it is worthwhile to compare the qualitative features of the excitation functions for deuteron and alpha particles as exhibited in Figs. 1, 2, and 3.

(d,n) and (α,n) reactions: The factor of about 3 between the (α,n) and (d,n) peak cross sections is not surprising because of the large (d,p) stripping cross section expected for deuterons, particularly at the lower energies. The stripping reactions evidently lower the compound-nucleus contribution to the (d,n) reaction by a greater amount than is added by (d,n) stripping.

TABLE III. Reactions of deuterons with titanium isotopes.

Product	Reaction
Sc^{47}	$Ti^{47}(d,2p)Sc^{47}$ $Ti^{48}(d,2pn \text{ or } dp)Sc^{47}$ (negligible up to E_d=20 MeV) $Ti^{49}(d,\alpha)Sc^{47}$ $Ti^{50}(d,\alpha n)Sc^{47}$
$Sc^{44,44m}$	$Ti^{46}(d,\alpha)Sc^{44,44m}$ $Ti^{47}(d,\alpha n)Sc^{44,44m}$
Sc^{46}	$Ti^{46}(d,2p)Sc^{46}$ $Ti^{47}(d,2pn \text{ or } dp)Sc^{46}$ (negligible up to E_d=20 MeV) $Ti^{48}(d,\alpha)Sc^{46}$ $Ti^{49}(d,\alpha n)Sc^{46}$
V^{48}	$Ti^{47}(d,n)V^{48}$ $Ti^{48}(d,2n)V^{48}$
V^{47}	$Ti^{46}(d,n)V^{47}$ $Ti^{47}(d,2n)V^{47}$

TABLE IV. Cross sections for deuterons with enriched titanium (mb) (results include contributions from other titanium isotopes).

(a) Ti^{46}

Run No.	E_d (MeV)	σ (Sc^{44m})	σ (Sc^{44g})	σ (Sc^{44m+g})	σ (Sc^{46})	σ (V^{47})
KC-6	20.2	4.3	11.0	15.4	101.2	
	19.4	4.3	11.8	17.2	91.4	
	17.5	8.0	15.8	23.8	76.6	
	15.6	11.2	25.8	37.0	63.8	
	13.8	12.2	32.7	45.0	40.4	
	12.0	11.3	40.7	52.1	18.2	
	9.6	10.3	55.2	65.5	10	
	7.7	4.2	32.4	36.7		
	6.7	2.3	22.9	25.2		
	5.5	1.2	13.6	14.8		
KC-9	20.2					65.6
	16.8					83.7
	13.0					143
	9.4					177
KC-10	18.5					68.8
	14.6					141
	11.0					261
	7.4					303

(b) Ti^{47}

Run No.	E_d (MeV)	σ (V^{48})	σ (V^{47})	σ (Sc^{47})	σ (Sc^{44m})	σ (Sc^{44})	σ (Sc^{44m+g})
KC-3	20.2	66.0		38.8	25.4	48.6	74.0
	19.5	69.8		38.4	25.2	47.8	73.0
	17.6	82.5		32.7	21.5	41.7	63.2
	15.7	91.8		27.4	16.0	32.9	48.9
	14.0	94.2		20.7	8.5	16.0	24.5
	12.1	108					
	9.8	114		4.40			
	8.0	178		1.52			
	7.0	156		0.73			
	5.8	140		0.53			
KC-8	18.6		371				
	14.7		372				
	11.1		267				
	7.5		36.9				
KC-11	20.1		203				
	16.7		232				
	12.9		257				
	9.3		123				
KC-15	17.7		342				
	13.7		347		6.50	11.4	17.9
	9.5		234				
	5.5		0.96				
KC-16	20.2		247	39.8	27.3	50.0	77.3
	15.7		278				
KC-17	20.2		254	38.6	26.9	46.2	73.1
	15.7		260				
KC-18	20.2			40.2	26.0	45.2	71.2
	15.9			23.4	15.5	28.9	44.4
	11.5			11.5	1.4	3.1	4.5
	4.2			0.60			

(c) Ti^{48}

Run No.	E_d (MeV)	σ (Sc^{46})	σ (V^{48})
KC-1	20.3		258
	17.7		334
	14.9	24.7	317
	12.1	41.2	256
	9.5	34.3	78.4
	6.9	23.8	
KC-2	15.9	17.9	
	13.3	39.2	
	10.8	40.6	144
	8.8	31.1	33.1

(d) Ti^{49}

Run No.	E_d (MeV)	σ (Sc^{47})	σ (Sc^{46})
KC-12	20.1	11.6	26.1
	17.4	18.4	25.3
	14.6	24.9	13.8
	12.0	41.6	6.8
	9.4	43.0	
	6.9	14.0	

(e) Ti^{50}

Run No.	E_d (MeV)	σ (Sc^{47})
KC-13	17.4	65.2
	14.6	36.0
	12.0	12.4
	9.4	2.5

The effect of stripping is also seen in the more gradual decrease with increasing excitation energy in the (d,n) excitation functions as compared to that for the (α,n) reactions. However, the observation that the peaks of the two excitation functions occur at closely the same excitation energies of the compound system V^{49}, suggests that, near the peak, the main contribution to the (d,n) reaction comes from a compound-nucleus mechanism.

$(d,\alpha n)$ and $(\alpha,\alpha n)$ reactions: In Fig. 1 it is seen that the cross section for the $(\alpha,\alpha n)$ reaction exceeds that for the $(d,\alpha n)$ reaction by a factor of form 2 to 3. From Fig. 3 it is evident that the difference appears mainly in the formation of $Sc^{44m}(I=7^+$ or $6^+)$ and to a lesser amount in the formation of $Sc^{44g}(I=3^+$ or $2^+)$. Again this speaks for the diminution of compound-nucleus reactions by competition from stripping because it is just the high angular-momentum transfers from the deuteron to a compound nucleus with which the stripping reactions will most successfully compete. This effect is also seen from the data for the $Ti^{46}(d,\alpha)Sc^{44m,44g}$ as shown in Fig. 5; again the large angular-momentum transfers by deuterons are suppressed.

There is also the possibility that a substantial direct-interaction contribution to the $(\alpha,\alpha n)$ reaction is the source of its large cross section relative to that for the $(d,\alpha n)$ reaction. This should not be a large part of the difference, though, as it would require the unlikely situation that the direct process transfer more angular momentum than the compound process.

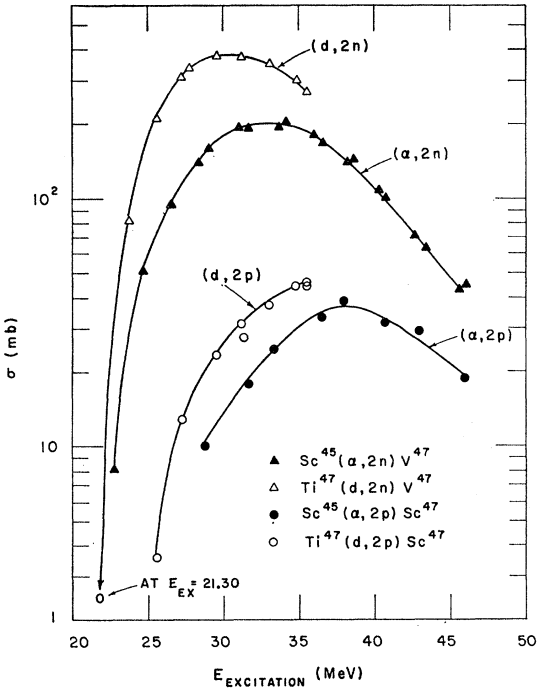

FIG. 2. Excitation functions for the $(d,2n)$, $(\alpha,2n)$, $(d,2p)$, and $(\alpha,2p)$ reactions.

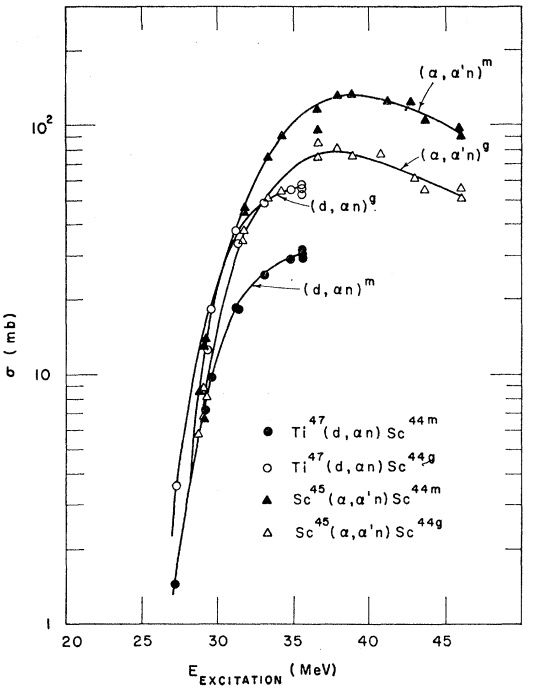

FIG. 3. Excitation functions for the $(d,\alpha n)^m$, $(d,\alpha n)^g$, $(\alpha,\alpha' n)^m$, and $(\alpha,\alpha' n)^g$ reactions.

$(d,2n)$, $(d,2p)$, $(\alpha,2n)$, and $(\alpha,2p)$ reactions: Here a different situation from that of the previous two sets of reactions is found: From Fig. 2 we see that the deuteron cross sections are larger than the corresponding alpha cross sections. In the spirit of the discussion up to this point, this behavior suggests that there is a significant contribution of the stripping process to both the $(d,2n)$

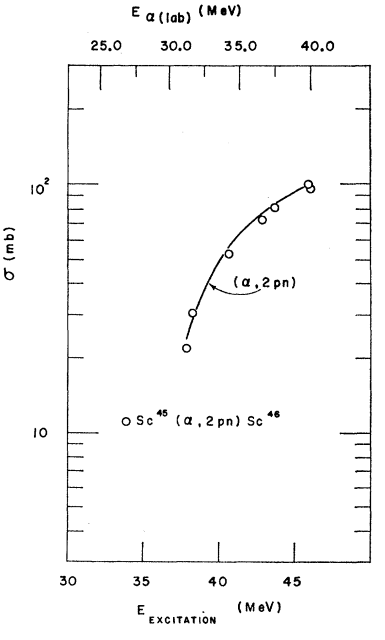

FIG. 4. Excitation function for the $(\alpha,2pn)$ reaction.

TABLE V. Cross sections for deuterons with titanium-47 (mb) (after correction for contributions from other titanium isotopes).

Run No.	E_d(MeV)	E_{ex}(MeV)	$\sigma(d,n)$	$\sigma(d,2n)$	$\sigma(d,2p)$	$\sigma(d,\alpha n)^m$	$\sigma(d,\alpha n)^g$	$\sigma(d,\alpha n)^{m+g}$
KC-3	20.5	35.4	42.0		44.5	29.6	56.6	86.2
	19.5	34.7	42.2		44.1	29.3	55.6	84.9
	17.6	32.9	51.9		37.5	25.0	48.4	73.4
	15.7	31.1	63.4		31.3	18.4	37.9	56.3
	14.0	29.4	70.1		23.6	9.7	18.0	27.7
	12.1	27.7	93.0					
	9.8	25.5	122.5		4.60		0.18	0.2
	8.0	23.7	208		1.48			
	7.0	22.7	182		0.70			
	5.8	21.6	164		0.61			
KC-8	18.6	33.8		432				
	14.7	30.1		432				
	11.1	26.7		308				
	7.5	23.2		38.5				
KC-11	20.1	35.3		236				
	16.7	32.1		270				
	12.9	28.4		297				
	9.3	24.9		140				
KC-15	17.7	33.0		398				
	13.7	29.1		403		7.31	12.5	19.8
	9.5	25.2		269				
	5.5	21.3		0				
KC-16	20.2	35.4		287	45.6	31.8	58.1	89.9
	15.7	31.1		323				
KC-17	20.2	35.4		285	44.3	31.3	53.7	85.0
	15.7	31.1		302				
KC-18	20.2	35.4			46.1	30.2	52.6	82.8
	15.9	31.3			26.7	17.9	33.2	51.1
	11.5	27.1			12.9	1.4	3.5	5.0
	4.2	20.1			0.70			

and $(d,2p)$ reactions. Since the energy carried in by the particle that is captured in the stripping process can lead to intermediate nuclei that are unstable with respect to particle emission, we expect a contribution to the $(d,2n)$ reaction by (d,n) stripping followed by evaporation of a neutron as well as the corresponding process for the $(d,2p)$ reaction.

Difficulties with this argument arise when the energy dependence of the relative cross sections for the $(d,2n)$ and $(d,2p)$ reactions are examined. While this question will be discussed at greater length below, it should be noted at this point that this ratio of cross sections behaves precisely as if the two reactions proceeded essentially entirely through compound-nucleus formation rather than through stripping. This behavior is shown in Fig. 6 where it is seen that the ratio of the cross sections of the $(d,2n)$ and $(d,2p)$ reactions is the same function of the excitation energy of the compound system, V^{49}, as it is for the $(\alpha,2n)$ and $(\alpha,2p)$ reactions. If either one or both of the two sets of reactions proceeded significantly through a direct-interaction mechanism there is no prior reason why the two ratio curves should bear any resemblance to each other.

TABLE VI. Cross sections for deuterons with titanium-46 (mb) (after correction for contributions from other titanium isotopes).

Run No.	E_d(MeV)	E_{ex}(MeV)	$\sigma(d,\alpha)^m$	$\sigma(d,\alpha)^g$	$\sigma(d,\alpha)^{m+g}$
KC-6	20.2	32.6	4.4	11.8	16.2
	19.4	31.9	5.7	12.8	18.5
	17.5	30.0	8.8	17.5	26.3
	15.6	28.2	12.7	29.3	45.0
	13.8	26.5	14.0	37.7	51.8
	12.0	24.7	13.1	47.2	60.4
	9.6	22.5	12.0	64.1	76.1
	7.7	20.7	4.9	37.7	42.6
	6.7	19.7	2.7	26.5	29.3
	5.5	18.5	1.3	15.8	17.2

Since there is no serious reason to assume that the alpha-particle reactions reported here proceed through any but a compound-nucleus mechanism [with the possible exception of $(\alpha,\alpha n)$ reactions at the higher energies], an attempt was made to reproduce the measured excitation function by the usual calculation based upon evaporation theory.[12] The values of the parameters required by this calculation were chosen to give maximum agreement between experiment and calculation for the alpha-particle excitation function, and were then used in an investigation of the relative contribution of

[12] J. B. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952).

compound and noncompound processes in the deuteron-induced reactions.

Evaporation calculations: A compound-nucleus reaction may be represented schematically as

$$a+X \rightarrow C \rightarrow b+Y,$$

where a and b are the incident and outgoing particles. X and Y are the initial and final nuclei, and C is the excited compound nucleus. Assuming the independence of formation and decay of the compound nucleus, the cross section for such a reaction can be expressed as

$$\sigma(a,b)=\sigma_C(\epsilon_a)W_b'(U)/\sum_j W_j(U), \quad (1)$$

where $\sigma_C(\epsilon_a)$ is the cross section for compound nucleus

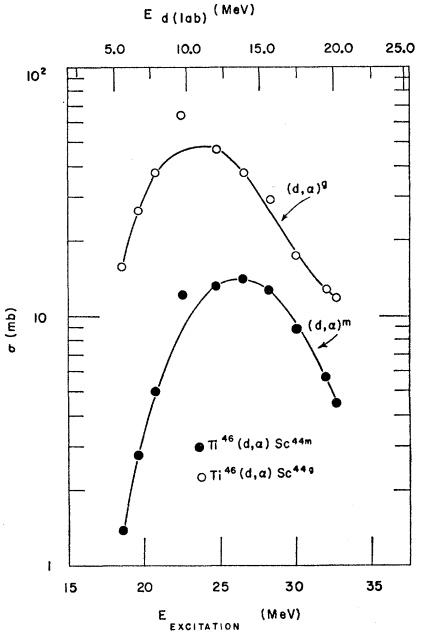

FIG. 5. Excitation functions for the $Ti^{46}(d,\alpha)Sc^{44,44m}$ reactions.

formation with ϵ_a as the kinetic energy of the incident particle, $W_b'(U)$ is the probability per unit time that the compound nucleus with excitation energy U emits particle b and no other, and $W_j(U)$ is the probability per unit time that the compound nucleus emits particle j. The summation includes all particles j that can be emitted.

According to the evaporation theory, the probability per unit time for emitting a particle with kinetic energy between ϵ and $\epsilon+d\epsilon$ is

$$P_j(\epsilon)d\epsilon=\frac{g_j m_j}{\pi^2\hbar^3}\sigma(\epsilon)\frac{\rho_f}{\rho_i}\epsilon d\epsilon, \quad (2)$$

where g_j=statistical weight of particle j; m_j=reduced mass of the system; $\sigma(\epsilon)$=cross section for the inverse

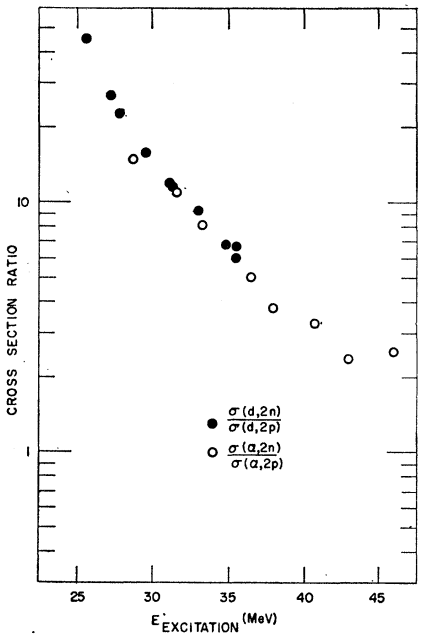

FIG. 6. Ratio curves for $\sigma(d,2n)/\sigma(d,2p)$ and $\sigma(\alpha,2n)/\sigma(\alpha,2p)$.

of the evaporation reaction; and ρ_i,ρ_f=level densities of the initial and final nuclei. In this calculation, the level density of a nucleus of mass A at excitation energy

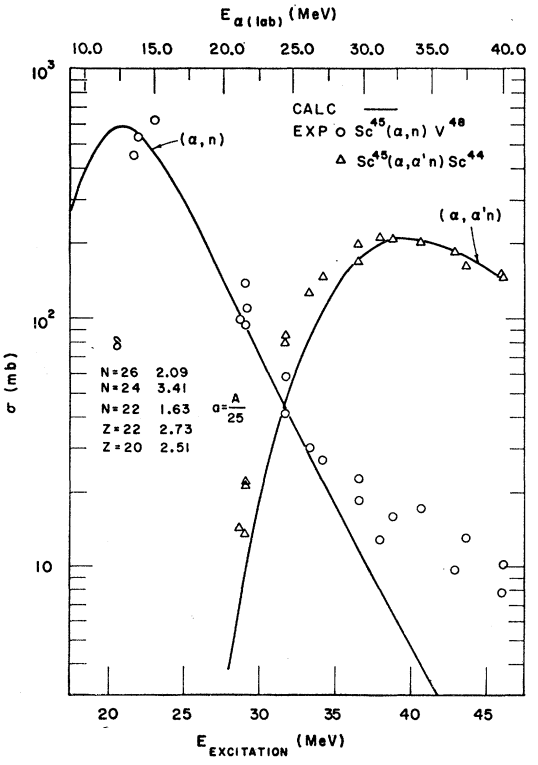

FIG. 7. Calculated excitation functions for the (α,n) and $(\alpha,\alpha'n)$ reactions.

U is taken as

$$\rho(U)=C\exp\{2[a(U-\delta)]^{1/2}\}. \quad (3)$$

The level density parameter "a" is assumed to be proportional to A; the constant "C" is assumed to be independent of A within a small range of A's. As was pointed out by Hurwitz and Bethe,[13] nuclear level density is not only dependent upon the total number of nucleons in the nucleus but also on whether or not the number of neutrons or protons is odd or even. The odd-even effect is incorporated into the level density formula by subtracting a quantity δ, the energy above the ground state of the "characteristic" level. This procedure is described in the work of Dostrovsky *et al.*[1] The level density of a nucleus, whose excitation energy is between ground level and δ, is taken to be the constant "C."

The empirical formulas of inverse cross sections of Dostrovsky *et al.*[1] are used for neutrons and for charged particles whose kinetic energy is at least 1.8 times the barrier height. For charged particles with kinetic energy less than 1.8 times the barrier height, Shapiro's[14] tables, based on accurate values of the Coulomb wave functions, are used to compute the inverse cross sections. A radius parameter of $r_0=1.7\times10^{-13}$ was used throughout.

The cross section for evaporation of one particle, based on the above assumptions, can be written as

$$\sigma(a,b)=\sigma_C(\epsilon_a)g_b m_b\left[\int_A^{U-S_b-\delta_b}\sigma(\epsilon_b)e^{2[a(U-S_b-\delta_b-\epsilon_b)]^{1/2}}\epsilon_b d\epsilon_b+\int_{U-S_b-\delta_b}^{U-S_b}\sigma(\epsilon_b)\epsilon_b d\epsilon_b\right]\Big/$$
$$\sum_j g_j m_j\left[\int_0^{U-S_j-\delta_j}\sigma(\epsilon_j)e^{2[a(U-S_j-\delta_j-\epsilon_j)]^{1/2}}\epsilon_j d\epsilon_j+\int_{U-S_j-\delta_j}^{U-S_j}\sigma(\epsilon_j)\epsilon_j d\epsilon_j\right], \quad (4)$$

where the lower limit of the first integral in the numerator "A" has the value $A=0$ if $U<S_b+S_2$, $A=U-S_b-S_2$ if $U>S_b+S_2$, and U=excitation energy of the compound nucleus; S_2=separation energy of the most loosely bound particle after the evaporation of b, $\sum_j$=summation over six of the possible particles that may be emitted, including b; proton, deuteron, alpha particle, tritons, He^3, neutron. S_j=separation energy of particle j. ϵ_j=kinetic energy of particle j. $\delta_j=\delta$ of the residual nucleus after the evaporation of particle j. For the evaporation of two particles, a similar expression can be written

$$\sigma(a,bc)=\sigma_c(\epsilon_a)\int_0^{U-S_b-S_c}P_b(\epsilon_b)W_c'(\epsilon_b)d\epsilon_b\Big/\sum_i\int_0^{U-S_j}P_j(\epsilon_j)d\epsilon_j, \quad (5a)$$

$$W_c'(\epsilon_b)=\int_{A'}^{U-S_b-S_c-\epsilon_b}P_c(\epsilon_c)d\epsilon_c\Big/\sum_k\int_0^{U-S_b-S_k-\epsilon_b}P_k(\epsilon_k)d\epsilon_k, \quad (5b)$$

where the lower limit A' is $A'=0$ for $U<S_b+S_c+\epsilon_b+S_3$, $A'=U-S_b-S_c-\epsilon_b-S_3$ for $U>S_b+S_c+\epsilon_b+S_3$, S_3=separation energy of the most loosely bound particle after the evaporation of both b and c.

Equations (4) and (5) were programmed for computation on an IBM-709 computer. With the computer programs and the parameters "a" and δ, a set of excitation functions was found that fit the experimental data from the reactions of alpha particles with Sc^{45}.

Alpha Reactions: The excitation functions for the alpha-induced reactions calculated from Eqs. (4) and (5) are compared with the experimental results in Figs. 7 and 8. Despite the rather small value of "a" that was used, the calculation was unable to reproduce the high-energy tail of the (α,n) excitation function. Both the small value of "a" and the discrepancy at higher energies are partly caused by the effects of angular momentum which were not included in this calculation. These effects may be approximately accounted for in the manner of Houck and Miller[15] by adjusting the quantities "S_2"; or, more accurately, by detailed calculations as described by Grover.[16] On the whole, though, the agreement between computed and measured values is satisfactory.

Two questions immediately arise in connection with this calculation: What is the significance of a success in a calculation of this type and what is the significance of the values of the parameters that are required for a successful fitting of the measured curves? It may be argued that although the formalism stems from the assumption of the formation of a compound nucleus whose behavior obeys the requirements of the statistical assumption, any success of the computation is a necessary but not a sufficient condition for a proof that these two assumptions are, indeed, correct. This is so because, regardless of mechanism, any reaction must have an apparent threshold governed by binding energies and Coulomb barriers and should also ultimately

[13] H. Hurwitz and H. A. Bethe, Phys. Rev. **81**, 898 (1951).
[14] M. M. Shapiro, Phys. Rev. **90**, 171 (1953).
[15] F. S. Houck and J. M. Miller, Phys. Rev. **123**, 231 (1961).
[16] J. R. Grover, Phys. Rev. **123**, 267 (1961).

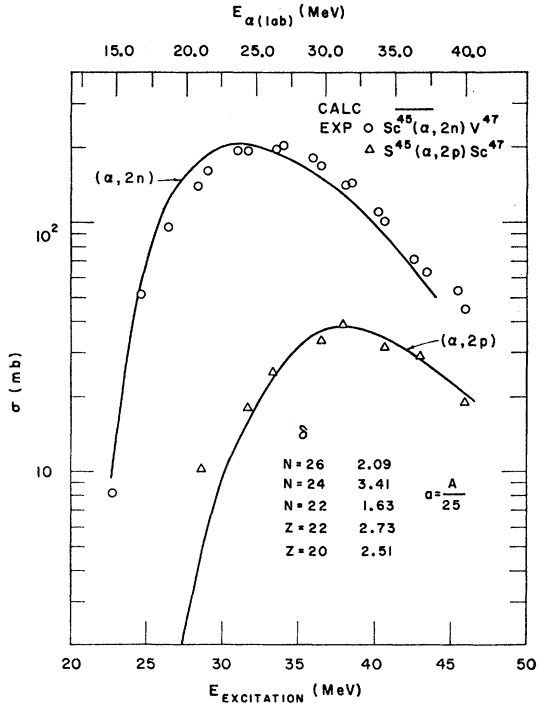

FIG. 8. Calculated excitation functions for the $(\alpha,2n)$ and $(\alpha,2p)$ reactions.

diminish in probability because of the onset of competing reactions. The maximum in the excitation function may be suppressed if the direct coupling between initial and final channels is strong enough to suppress the effects of other competing channels that are energetically possible. To put it another way, the excitation functions that are calculated in this manner depend primarily upon the volume of phase space available to the products and will thus be indistinguishable from any other formalism which depends mainly upon available phase space and whose matrix elements have an energy dependence that is approximately represented by the product of channel velocity and a transmission coefficient. Thus it is unlikely that any reaction proceeding to an appreciable extent through a direct interaction as described, for example, by Butler *et al.*[17] would have an excitation function that could be fitted by Eq. (1). It may be said, then, that success in this type of calculation can only show that certain extreme direct-interaction mechanisms make a negligible contribution to the reaction in question; but departures from the statistical assumption that could have important effects upon angular distributions and widths for emission of particular particles might still not distort excitation functions so drastically that they could not be described by the formalism used here.

The parameters that are available within this theory are those in the level-density formula (3): "a" and "δ." In the region of mass number where the Coulomb barriers are not very large, the parameter "a" mainly affects the shape of the excitation functions and the parameter "δ" mainly affects their magnitude (if no account is taken of the states between the ground state and the characteristic state, "δ," then "δ" will also affect the apparent threshold of a reaction and the position of the peak of its excitation function). As was pointed out earlier, since the angular momentum of the compound nucleus is of significance for its emission processes, and since the dependence of level density upon angular momentum was not included in this calculation, the value of "a" will be perturbed in such a direction as to compensate for these effects. Angular momentum effects will tend to make the values of "a" too large when determined from the energy spectrum of emitted particles, and, as discussed by Grover,[16] will tend to make the value of "a" too small when determined from the analysis of excitation functions.

The quantities, "δ" as was stated before, are not uniquely determined in this calculation; the calculation is more sensitive to their differences than to their absolute values. But this complication aside, the quantities "δ" should be viewed as parameters that are required to describe the distribution of bound states between ground and continuum, as well as unbound states, with a simple continuous function as in (3) and therefore related to, but not necessarily equal to, such stationary state properties as pairing energies and energy gaps.

It is of particular interest to see from Fig. 7 that the calculation can satisfactorily reproduce the excitation function for the $(\alpha,\alpha n)$ reaction, a reaction that might be expected to have a substantial direct-interaction contribution. The broad maximum, as explained before,[1] occurs in the calculation because the observed excitation function is in fact the sum of two excitation functions, those for the $(\alpha,\alpha n)$ and the $(\alpha,n\alpha)$ reactions, which will, in general, have different shapes and peak positions. Indeed, in this calculation, the values of "δ" that were required to give proper magnitudes for both the V^{48} and Sc^{44} yields, were such that the $(\alpha,n\alpha)$ reaction dominated the $(\alpha,\alpha n)$.

Deuteron Reactions: The reactions of deuterons with Ti^{47} were investigated in order to examine further the range of usefulness of the compound-nucleus model. These reactions proceed through the same compound system as that in the reactions of alpha particles with Sc^{45}; thus by comparing the excitation functions from these two sets of reactions, not only the independence of the modes of formation and decay of the compound nucleus may be tested, but also the importance of noncompound processes in the deuteron induced reactions may be estimated.

The excitation functions for (d,n), $(d,2n)$, $(d,2p)$, and $(d,\alpha n)$ reactions were calculated with the same param-

[17] S. T. Butler, N. Austern, and C. Pearson, Phys. Rev. **112**, 1227 (1958).

eters that described the alpha induced reactions. Both the experimental and calculated results are shown in Figs. 9 and 10. When compared with the agreement obtained for the alpha-induced reactions, these calculations are very poor indeed. The divergence may arise from two sources: (1) The cross section for the formation of a compound nucleus is less than that predicted by continuum theory because of competition of noncompound processes, and (2) these noncompound processes also contribute to the observed cross sections.

It is expected that the well-known deuteron-stripping reactions are of importance and thus the cross section for the formation of a compound nucleus would be diminished. As shown in Fig. 1, the maximum cross section for the (d,n) reaction is only about one-third of that for the (α,n) reaction. If it is assumed that both the (α,n) and the (d,n) reactions are compound processes at these energies, then, since the continuum-theory cross sections are both about 800 mb, one would conclude that stripping reactions account for about 500 mb out of about 800 mb inelastic cross section. This estimate takes the cross section that is calculated by continuum theory as a measure of the total inelastic cross section for deuterons which includes stripping as well as absorption of the deuteron as a whole. The small cross section of about 300 mb that remains for compound-nucleus formation indicates that the effect of the Coulomb barrier upon compound-nucleus formation by deuterons

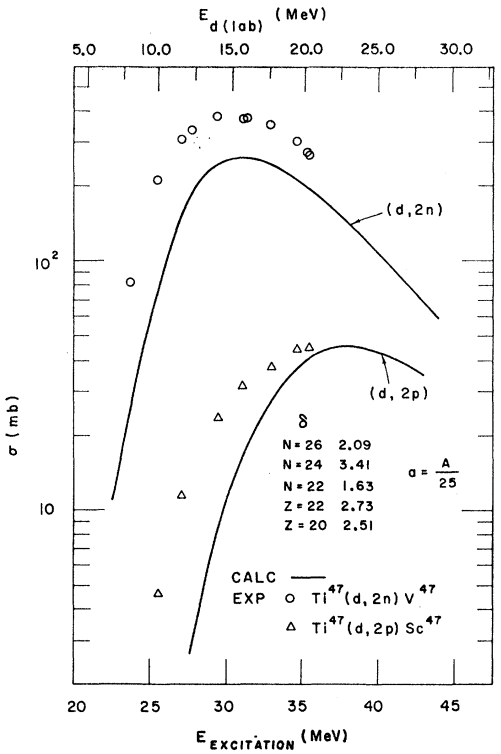

FIG. 10. Calculated excitation functions for the $(d,2n)$ and $(d,2p)$ reactions.

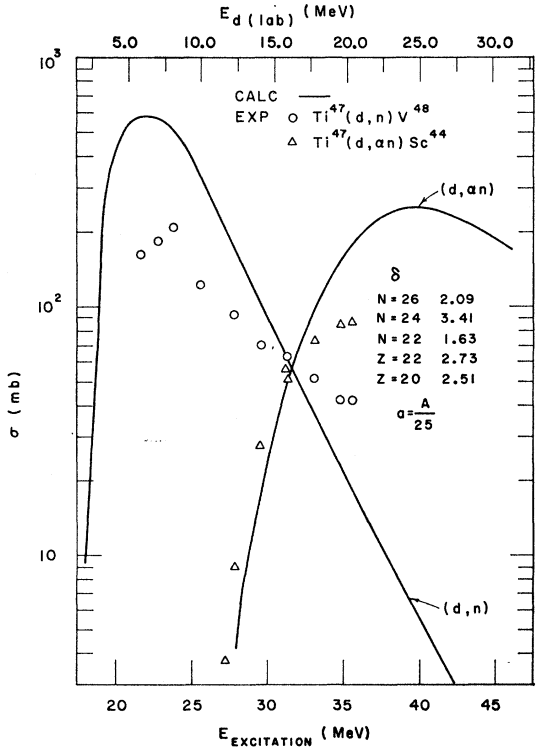

FIG. 9. Calculated excitation functions for the (d,n) and $(d,\alpha n)$ reactions.

is more closely that of the Coulomb interaction between the target nucleus and a free proton with half the kinetic energy of the deuteron rather than the interaction with a single charge of the energy and spatial distribution of the deuteron.

A comparison between the excitation functions of the $(\alpha,\alpha' n)$ and $(d,\alpha n)$ reactions can give an estimate of the upper limit of the cross section for the formation of the compound nucleus by deuterons in the higher energy region if it is assumed that both the $(\alpha,\alpha' n)$ and the $(d,\alpha n)$ reactions are compound processes up to excitation energy of 36 MeV. At an excitation energy of 35.5 MeV the cross section of the $(\alpha,\alpha' n)$ reaction is about 13% of the continuum-theory inelastic cross section. Since at this excitation energy the cross section of the $(d,\alpha n)$ reaction is 88 mb, the cross section for compound-nucleus formation by deuterons would be about 88/0.13 or 670 mb. This estimate is only a lower limit because of the possibility of a noncompound contribution to the $(\alpha,\alpha n)$ reaction.

It is evident, though, from the data illustrated in Fig. 1 that the (d,n) reaction is not completely a compound-nucleus one; the (d,n) excitation function does not diminish as rapidly as that for the (α,n) reaction. It is thus of interest to attempt to estimate the cross section of (d,n) stripping reaction using Peaslee's[18] analysis.

[18] D. C. Peaslee, Phys. Rev. 74, 1001 (1948).

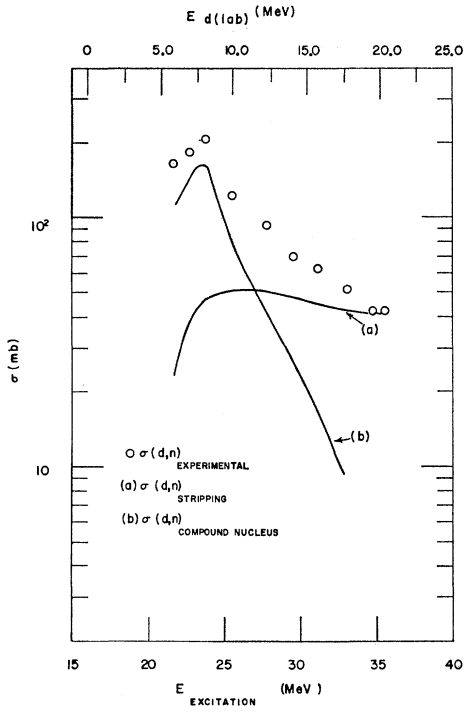

FIG. 11. Stripping and compound-nucleus contributions to the (d,n) reaction.

Although his method is only semiquantum mechanical, and involves many gross approximations, it is the only explicit formulation available on this subject. The values computed by his formulation, normalized to give the proper value at high energies, are shown in curve a of Fig. 11. At low energies the Coulomb barrier is effective in preventing the proton from approaching the nuclear surface, hence the (d,n) stripping cross section is low. However, at higher energies, the Coulomb barrier is less effective, and there is the gradual rise of curve a. After subtracting the calculated values from the experimental results, the estimated compound-nucleus contribution to the (d,n) reaction, curve b, is obtained. The shape of curve b certainly speaks strongly for a compound-nucleus process.

From the observation, illustrated in Fig. 2, that the $(d,2n)$ and $(d,2p)$ cross sections are consistently higher than those of the $(\alpha,2n)$ and $(\alpha,2p)$ reactions, it is evident that noncompound processes must also contribute substantially to these two deuterons reactions. If it is assumed that both the $(\alpha,2n)$ and the $(\alpha,2p)$ reactions proceed largely through the formation of a compound nucleus, then, since the cross section for compound nucleus formation by deuterons is no greater than that for alpha particles, the noncompound contributions to the $(d,2n)$ and $(d,2p)$ is at least $\frac{1}{3}$ at 35-MeV excitation. It is probably greater than $\frac{1}{3}$ because, as was discussed above, the deuteron capture cross section is probably smaller than that for alpha particles. The noncompound contribution to the $(d,2n)$ reaction is expected to come from (d,n) stripping followed by evaporation of a neutron, while that to the $(d,2p)$ reaction is expected to come from (d,p) stripping followed by evaporation of a proton.

Since the $(d,2n)$ and $(d,2p)$ reactions seem to have significant noncompound contributions, Bohr's assumption should not be supported: at the same excitation energy there is no reason to expect the ratio $\sigma(d,2n)/\sigma(d,2p)$ to be the same as that for $\sigma(\alpha,2n)/\sigma(\alpha,2p)$. However, the experimental ratios, as given in Fig. 6, show that $\sigma(d,2n)/\sigma(d,2p)$ is approximately equal to $\sigma(\alpha,2n)/\sigma(\alpha,2p)$ over the entire energy interval where they may be compared. This paradoxical result may be just the accidental consequence of the binding energies and Coulomb barriers in this particular situation leading to similar results for both the compound and noncompound $(d,2n)$ and $(d,2p)$ reactions. The investigation of this possibility requires formulations for the energy dependence of the relative probabilities of (d,p) and (d,n) stripping as well as for the spectra of neutrons and protons stripped from deuterons with energies between 5 and 20 MeV; such formulations unfortunately do not exist. The generality of the phenomenon is being investigated by measuring similar ratios from other compound systems.

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